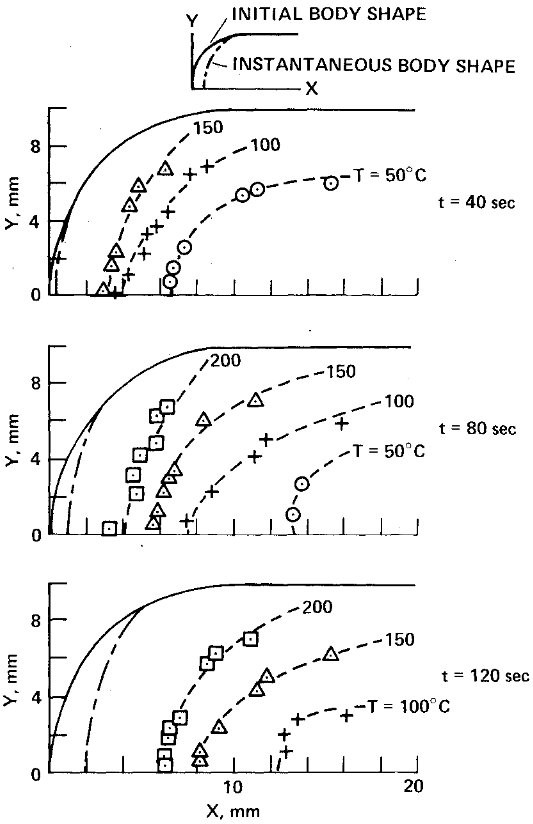

Fig. 3 Instantaneous internal temperature distribution.

thermal expansion remains as a permanent deformation due to the physical nature of Teflon. Therefore, the second method was adopted in this experiment. The effect of the thermal expansion was estimated by the following method: Before the wind tunnel is turned on, several axial positions are marked along the model. Displacements of the marks were measured from the photographs, and the value of the thermal expansion was evaluated approximately by extrapolating these data.

Results and Discussion

Figure 1 shows typical recession depth results. Since the nose is initially spherical, the initial x coordinate of the model is positive at any y coordinate except at the stagnation point ($y=0$). (Figure 1 indicates only an apparent value.) When the model at room temperature is suddenly exposed to a uniform high-enthalpy stream, large heat transfer takes place because of the large temperature difference. However, this heat is absorbed by the thermal capacity of the model itself and the surface temperature increases only slightly. Even though ablation occurs, the recesssion depth does not seem to be considerable at first. Therefore, the model length first increases because of thermal expansion.

Figure 2 shows the modified recession depth at the stagnation point compared with raw data. The figure shows that the recession rate attains steady-state conditions after 100 sec. The recession rate with consideration of thermal expansion, however, differs significantly from one which does not consider it. Previous experiments of steady ablation have neglected the effect of thermal expansion and, consequently, have underestimated the ablation rate.

The main purpose of the present experiment was to determine the transient ablation from the point of view of the instantaneous internal temperature distribution. Figure 3 shows the instantaneous isothermal lines at intervals of 50°C; the internal temperature increases with time and the local thermal expansion cannot be neglected. The internal temperature distribution changes abruptly in a very thin layer, i.e., gel layer,[5] near the body surface. The thickness of this layer is of the same order as that of the thermocouple junction, so that the measurements in the surface region are not reliable. Furthermore, the diffusion of heat inside the ablator in the direction parallel to the surface tangent cannot be neglected.

Conclusions

It takes more than 60 sec for recession to exceed the thermal expansion at stagnation point, and the effect of the thermal expansion on recession velocity cannot be neglected. Note that the heat flow inside the heat-shield material in the direction parallel to the surface tangent cannot be neglected. Therefore, a one-dimensional analysis is not valid.

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Intermittent Transition Flow in a Boundary Layer

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Introduction

DHAWAN and Narasimha[1] developed a method of calculating the properties of a boundary layer undergoing transition by preserving the essential intermittency of the flow. Narasimha[2] modified Emmons'[3] original formulation to obtain an intermittency function described by

$$\gamma = 1 - e^{-A\xi^2} \tag{1}$$

where $\xi = (x - x_t)/\lambda$, x_t is the transition point, and A and λ are empirical constants. Using the data of Schubauer and Klebanoff,[4] A was evaluated as 0.412; λ is a fit factor defined as

$$\lambda = (x)_{\gamma=0.75} - (x)_{\gamma=0.25} \tag{2}$$

By comparison with other data Eq. (1) was shown to be a good approximation to a "universal" intermittency function for boundary-layer transition. Some ambiguity did appear near $\gamma = 0$ which was attributed mainly to the influence of pressure gradients. In order to generalize this approach, a

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correlation between λ and the transition point was deduced as

$$Re_\lambda = 5\ (Re_t)^{0.8} \tag{3}$$

The data from which this expression was developed showed a wide scatter, which the authors attributed to "unspecified differences in experimental conditions and a possible dependence of λ on the agencies causing transition." Knowing the intermittency distribution, subject to the specification of the transition Reynolds number, it can be shown[1] that the mean velocity profile during transition is given by

$$\bar{u} = (1-\gamma)u_L + \gamma u_T \tag{4}$$

and that the local skin friction coefficient is

$$c_f = (1-\gamma)c_f^L + \gamma c_f^T \tag{5}$$

The indices L and T indicate quantities appropriate to fully developed laminar or turbulent flow if it were to exist at the local value of length Reynolds number.

This problem is re-examined here from a more fundamental point of view. All results are derived directly from the equations of motion using the concept of the conditioned average developed theoretically by Libby.[5] The theory does not contain any empiricism other than that inherent in the normal analysis of boundary layers. It is a simple integral approach that reduces to a first-order, ordinary differential equation for intermittency as a function of length Reynold's number.

Conditioning Function

Following Libby,[5] the conditioning function $I = I\ (x,t)$ is defined to be one if the point x at the time t is covered by turbulent flow, and zero if it is covered by laminar flow. This can be given physical significance by considering a mass balance for the fluid composing those turbulent spots passing through an arbitrary, stationary control volume in the flow. Defining a source function $w\ (x,t)$, taken to represent the rate of generation of turbulence per unit mass, a continuity equation for the incompressible fluid comprising the turbulent spots may be derived:

$$\frac{\partial I}{\partial t} + \frac{\partial I u_k}{\partial x_k} = \dot{w} \tag{6}$$

It follows that the time average of this source term is given by

$$\bar{\dot{w}} = \bar{u}_k \frac{\partial \gamma}{\partial x_k} \tag{7}$$

where γ is the intermittency function defined by

$$\gamma(x) = \frac{1}{T}\int_0^T I(x,t)\,\mathrm{d}t \tag{8}$$

and $\bar{u}_k$ is given by Eq. (4). This result indicates that, as a fluid particle that contains a region of localized turbulence is convected by the mean flow, the intermittency at a point will increase if the mass of turbulent fluid within the particle increases.

Equation for Intermittency

The basic approach is to average the equations of motion conditionally, first multiplying by I and then by $(1-I)$ and time-averaging. The details of this are discussed at some length in the paper by Libby.[5] After making the boundary-layer assumptions, this results in continuity and momentum equations for both the laminar and turbulent phases, which together with (7) may be viewed as five equations in the five unknowns u^T, v^T, u^L, v^L, and γ. These equations are

$$\gamma\left(\frac{\partial u^T}{\partial x} + \frac{\partial v^T}{\partial y}\right) + (1-\gamma)(u^T - u^L)\frac{\partial \gamma}{\partial x} = 0 \tag{9}$$

$$(1-\gamma)\left(\frac{\partial u^L}{\partial x} + \frac{\partial v^L}{\partial y}\right) + \gamma(u^T - u^L)\frac{\partial \gamma}{\partial x} = 0 \tag{10}$$

$$\gamma\left(u^T\frac{\partial u^T}{\partial x} + v^T\frac{\partial u^T}{\partial y}\right) + u^T[\gamma u^T + (1-\gamma)u^L]\frac{\partial \gamma}{\partial x} + \frac{\partial}{\partial y}\widetilde{u'v'} = \gamma\nu\frac{\partial^2 u^T}{\partial y^2} + u\bar{\dot{w}} \tag{11}$$

$$(1-\gamma)\left(u^L\frac{\partial u^L}{\partial x} + v^L\frac{\partial u^L}{\partial y}\right) - u^L[\gamma u^T + (1-\gamma)u^L]\frac{\partial \gamma}{\partial x} = (1-\gamma)\nu\frac{\partial^2 u^L}{\partial y^2} - u\bar{\dot{w}} \tag{12}$$

Consider that by adding Eqs. (11) and (12) the coupling term cancels, and supressing Eq. (8), as an equation for determining $\bar{\dot{w}}$, leaves three equations. Two additional relationships among the dependent variables are required. These are provided by profile approximations for u_T and u_L.

The assumed profiles are shown on Fig. 1. The laminar profile is approximated by the linear viscous sublayer law, which may be thought of as representing the leading term in the Blasius series. The turbulent profile is represented by Spalding's law-of-the-wall. Both of these profiles must be scaled with respect to the total skin friction as given by Eq. (5) rather than being individually scaled by local turbulent or laminar values. Hence they depend parametrically on intermittency through skin friction ratios. Figure 1 also demonstrates the validity of the conditional average when applied to the mean velocity by comparison with the data of Schubauer and Klebanoff.[4]

Substituting these profile approximations into the preceding equations and integrating across the boundary layer gives an equation for intermittency in the form

$$G_1\frac{\mathrm{d}\gamma}{\mathrm{d}Re_x} + G_2 = 0 \tag{13}$$

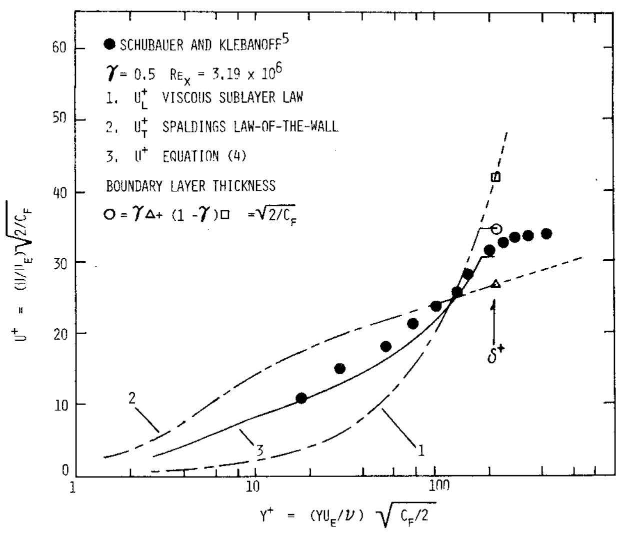

Fig. 1 Component velocity profiles.

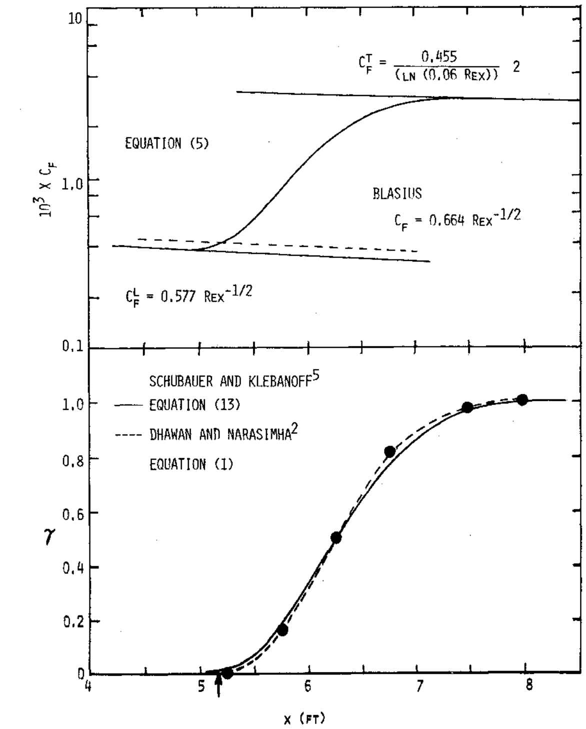

Fig. 2 Variation of intermittency and local skin friction through the transition region.

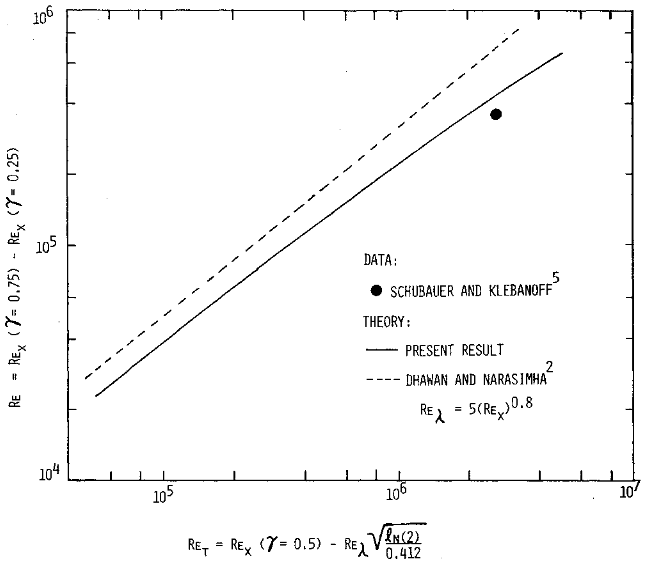

Fig. 3 Correlation between extent of the transition zone and the transition point.

The coefficients G_1 and G_2 depend upon the assumed velocity profiles, quadratures over these profiles, the assumed skin friction correlations, and their gradients, as well as the intermittency and the boundary-layer thickness.

When γ is constant at either zero or one, Eq. (13) reduces to a differential equation for skin friction. Use of these skin friction correlations in the present theory assures that the intermittency function will behave smoothly. The turbulent expression thus derived was obtained first by White,[6] and the laminar expression is approximately 10% low when compared to the exact Blasius formula (see Fig. 2).

Comparison with Experiment

The remarkable agreement between the present theory and the data of Schubauer and Klebanoff[4] is shown on Fig. 2. Also shown on this figure is the intermittency distribution deduced by Dhawan and Narasimha.[1] Clearly both of these results are in excellent agreement.

Figure 3 shows a plot of the correlation between Re_λ and Re_t and the same calculation using the present theory. Because of the asymptotic behavior of the theory near $\gamma = 0$, it is necessary to adopt an arbitrary definition of the transition point in order to generate these results. This is accomplished by solving Eq. (1) for Re_t having first calculated Re_λ and the value of Re_x corresponding to $\gamma = 0.5$. The result of this definition of Re_t for the data of Schubauer and Klebanoff is shown by an arrow on Fig. 2.

It is interesting to speculate as to the significance, if any, of this result. If it is true that the final stage of transition is independent of causitive disturbances, then incorporating different physical effects such as compressibility or pressure gradients in the equation of motion should produce a family of curves on a plot such as Fig. 3. Thus the precise nature of the distrubances causing transition would be incorporated only vestigally insofar as they affect the transition point.

Some experimental evidence is available in support of this idea provided by the work of Mitchner[7] and Schubauer and Klebanoff,[4] who showed that the growth of turbulent spots is identical whether they are artificially stimulated or occur naturally. If this is not the case, then a further parameterization would be required, based on the exact nature of the agencies causing transition. The data used by Dhawan and Narasimha[1] in no way shed light on this point. Having established the functional dependence of γ on length Reynolds number, it is possible to calculate accurately the corresponding variation in local skin friction from Eq. (5), as was demonstrated by Dhawan and Narasimha.[1]

Conclusions

The approach presented in this paper has been shown to be in good agreement with experimental data. These encouraging results, together with those of Libby,[5] suggest that the concept of the conditioned average is appropriate as a theoretical tool for analyzing intermittent boundary-layer flows. Also, the "averaging rule" developed by Dhawan and Narasimha may be generally useful for correlating transition data and data arising from other types of intermittent flows.

Because of the simplicity of the concepts used here, it should be possible to include the effects of other parameters such as pressure gradients in the present theory, since these are incorporated easily in the equation of motion. This would provide a new way of looking at other problems such as intermittent pipe flows, bluntbody flows, and relaminarizing flows.

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